# A Bayesian Model to Estimate Male and Female Fertility Patterns at a Subnational Level

Riccardo Omenti [1]   Monica Alexander [2]   Nicola Barban [1]

[1]University of Bologna

[2]University of Toronto

June 13, 2024

# Importance of measuring male fertility

- Male fertility measurement is overlooked (Coleman 2000)
- Increasing involvement of men in fertility decisions (Lappegård et al. 2011)
- Trajectories in male fertility can differ systematically from those of women due to:
  - **different reproductive age spans** (Schoumaker 2019)
  - **unbalanced sex ratios** (Dudel & Klüsener 2016)
  - **distinct cultural norms** (Dudel & Klüsener 2021)

# Objectives

- **Goal** $\rightarrow$ construction of a Bayesian model to estimate **male** and **female period TFR** at a **subnational** level
  - Essential component of population change
  - Shape local policies
- **Data example** $\rightarrow$ **US counties** during the **period 1982-2019**
  - High heterogeneity in fertility behaviors across time and space
  - High quality data registration systems

# Challenges

- Data on births disaggregated by **paternal ages** are often unavailable
  - Countries with **inefficient data registration systems** or lacking **high quality surveys**
  - Small regions with **masked ages** at **childbearing** due to **privacy concerns**
- Even in developed countries, birth registration systems have started recording childbearing ages of men quite recently
- The share of births with missing paternal ages is much higher than for maternal ages (Dudel & Klüsener 2016)

## Methodological Framework

Build on the **Bayesian model** by Schmertmann & Hauer (2019)

**Idea** $\rightarrow$ Estimation of period Total Fertility Rate (TFR) without knowledge of births by maternal ages

**Data Requirements**

- **Counts of children under 5**
- **Counts of women aged 15-49**

**Prior requirement**

- **Child mortality estimates**
- **Standard age-specific fertility schedules**

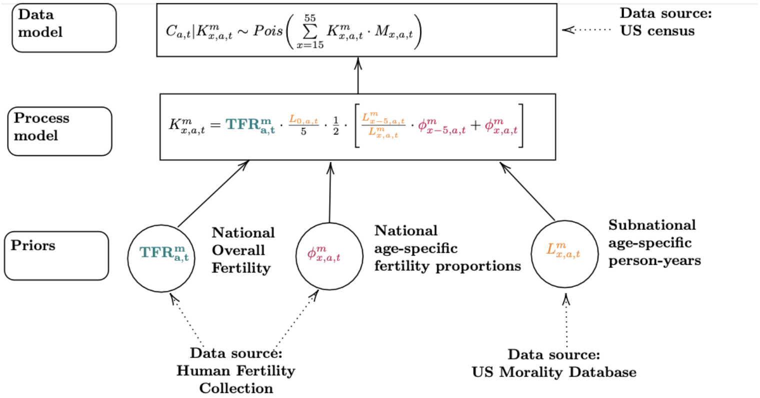
# Proposed extension

Extend the previous **Bayesian model** to estimate **male** and female **fertility** at a **subnational level**

**Extensions**

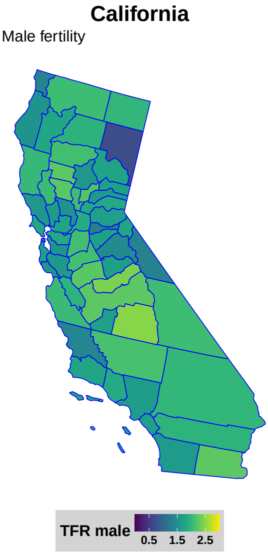
- Inclusion of **men aged 15-59**
- Incorporation of **subnational mortality estimates**
- Account for spatial dependencies → **information pooling**
- Account for temporal dependencies → **temporal smoothing**

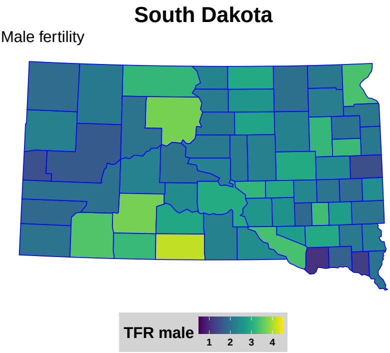
# Bayesian model summary



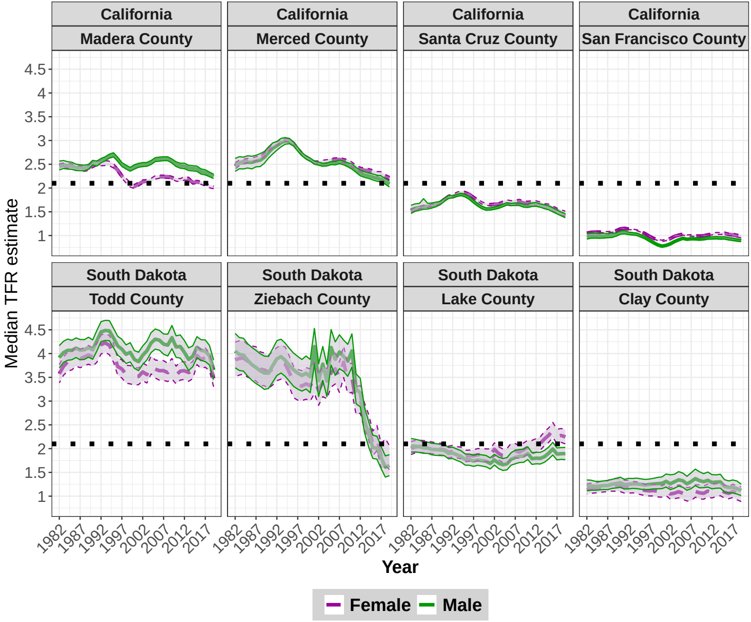

**Final Goal**$\rightarrow$ Draw samples from the **marginal posterior distribution** $TFR^s_{a,t}|$data, other parameters

# California and South Dakota in 2015



# Male and female TFR in selected counties



# Preliminary conclusions

- Using **county-level population counts** by age and sex allows to derive **subnational** period TFR estimates without the need of information on parental ages.
- **Male** and **female fertility** tend to converge by the end of the considered period
- Country-specific characteristics determine a high **spatial heterogeneity** and **distinct temporal trajectories**.

# Any Questions??

Looking forward to your feedback!

Contact: riccardo.omenti2@unibo.it

romenti.github.io @OmentiRiccardo

**Acknowledgements**: ERC Grant GENPOP n. 865356

# Essential Bibliography


Carl P Schmertmann and Mathew E Hauer.
Bayesian estimation of total fertility from a population's age–sex structure.
*Statistical Modelling*, 19(3):225–247, 2019.

David A Coleman.
*Male fertility trends in industrial countries: Theories in search of some evidence*.
International Union for the Scientific Study of Population, 1995.

Christian Dudel and Sebastian Klüsener.
Estimating male fertility in eastern and western germany since 1991: A new lowest low?
*Demographic Research*, 35:1549–1560, 2016.

Bruno Schoumaker.
Male fertility around the world and over time: How different is it from female fertility?
*Population and Development Review*, pages 459–487, 2019.

Christian Dudel, Yen-hsin Alice Cheng, and Sebastian Klüsener.
Shifting parental age differences in high-income countries: Insights and implications.
*Population and Development Review*, 49(4):879–908, 2023.

Li Zhang.
*Male fertility patterns and determinants*, volume 27.
Springer Science & Business Media, 2010.

# Bayesian model

**Data model**:

$$C_{a,t} | K^s_{x,a,t} \sim \text{Pois}\left( \sum_{x=15}^{\omega^s} K^s_{x,a,t} \cdot E^s_{x,a,t} \right)$$

$$K^s_{x,a,t} = TFR^s_{a,t} \cdot \frac{\tilde{L}_{0,a,t}}{5} \cdot \frac{1}{2} \cdot \left[ \frac{\tilde{L}^s_{x-5,a,t}}{\tilde{L}^s_{x,a,t}} \cdot \phi^s_{x-5,a,t} + \phi^s_{x,a,t} \right]$$

with $w^F = 45$ and $w^M = 55$

- **Overall fertility** ($TFR^s_{a,t}$)
- **Age- and sex-specific fertility proportions** ($\phi^s_{x,a,t}$)
- **Age- and sex-specific person-years** ($L^s_{x,a,t}$)

# Priors on fertility parameters

## Prior on TFR

$$TFR_{a,t}^{s} \sim \mathcal{N}(TFR_t^{nat,s}, \sigma^2_{TFR_{a,t}^{s}})$$

## Prior on age-specific fertility patterns

$$\gamma_{x,a,t}^{s} = m_x^s + y_{1,x}^s \beta_{1,a,t}^s + y_{2,x}^s \beta_{2,a,t}^s$$

$\gamma_{x,a,t}^{s} = \log\left(\frac{\phi_{x,a,t}^{s}}{\phi_{15,a,t}^{s}}\right)$

## Pooling information over countries

$$\beta_{p,a,t}^{s} \sim \mathcal{N}(\mu_{\beta_{p,t}^{s}}, \sigma^2_{\beta_{p,a,t}^{s}})$$

## Smoothing over time

$$\mu_{\beta_{p,t}^{s}} \sim \mathcal{N}(2\mu_{\beta_{p,t-1}^{s}} - \mu_{\beta_{p,t-2}^{s}}, \sigma^2_{\mu_{\beta_{p,t}^{s}}})$$

# Priors on mortality and standard deviation parameters

**Prior on Person-years**

$$\tilde{L}_{0,a,t} \sim \mathcal{N}(\hat{L}_{0,a,t}, \hat{\sigma}^2_{\hat{L}_{0,a,t}})$$

$$\tilde{L}^s_{x,a,t} \sim \mathcal{N}(\hat{L}^s_{x,a,t}, \hat{\sigma}^2_{\hat{L}_{0,a,t}})$$

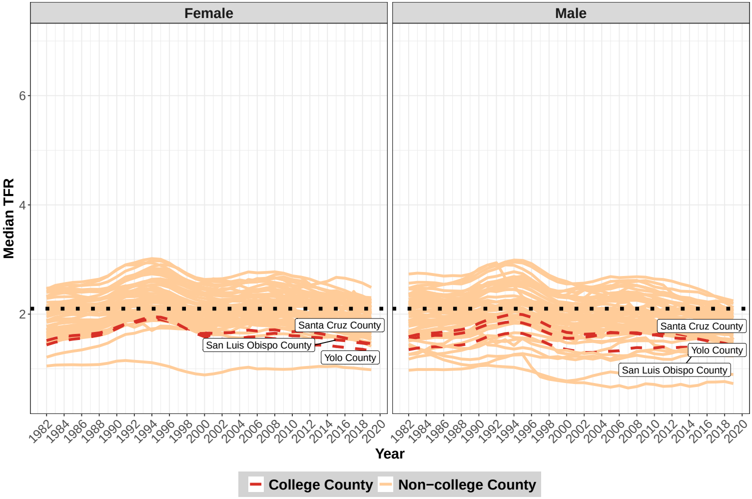
- $\tilde{L}^s_{x,a,t}$ from age-, period- and sex-specific subnational life tables
- Variances calculated from the standard errors available from the subnational life tables

**Prior on standard deviation parameters**

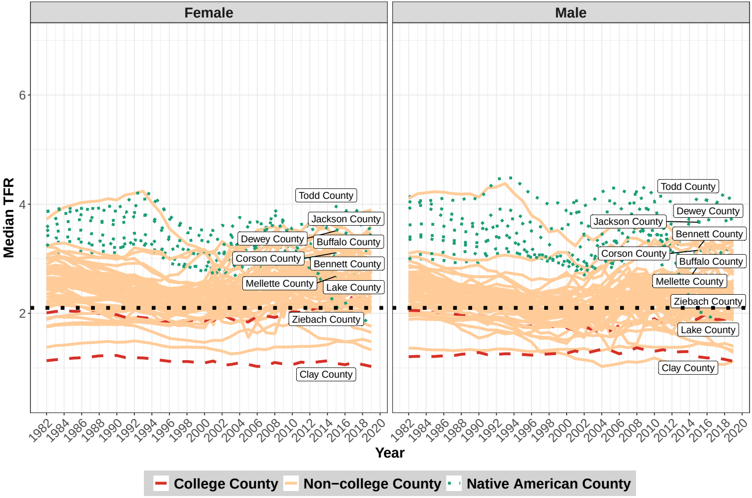
$$\sigma_{\beta^s_{p,a,t}}, \sigma_{TFR^s_{a,t}}, \sigma_{\mu_{\beta^s_{p,t}}} \sim \mathcal{N}^+(0,1)$$

- weakly informative priors

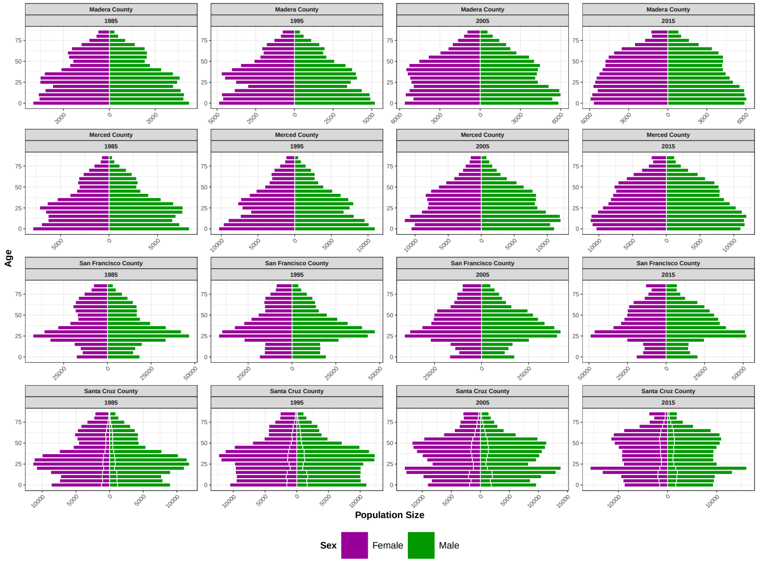
# Appendix: California TFR trajectories



# Appendix: South Dakota TFR trajectories



# Appendix: population pyramids (1)



# Appendix: population pyramids (2)